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Numerical Studies of Hall Thruster Acceleration Region Electron Transport

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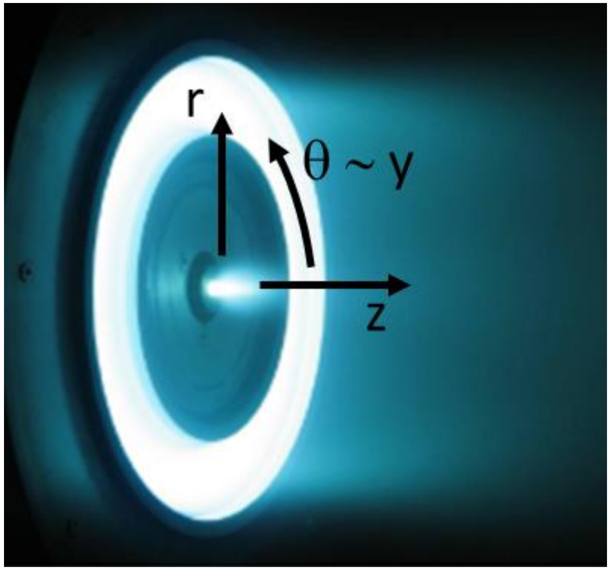
Numerical Studies of Hall Thruster Acceleration Region Electron Transport

- Straightforward implementation of quasi-linear wave growth models in Hall2De have had limited success
  - Our most successful attempt **turned off scattering by ion-acoustic waves when electrons are no longer Maxwellian**

    A. Lopez Ortega, I. Katz, V. Chaplin, “A First-Principles Model Based on Saturation of the Electron Cyclotron Drift Instability for Electron Transport in Hydrodynamics Simulations of Hall Thruster Plasmas”, IEPC 2017-178
- 1-D Azimuthal simulation of the acceleration region
- 2-D z-θ simulation of the acceleration region

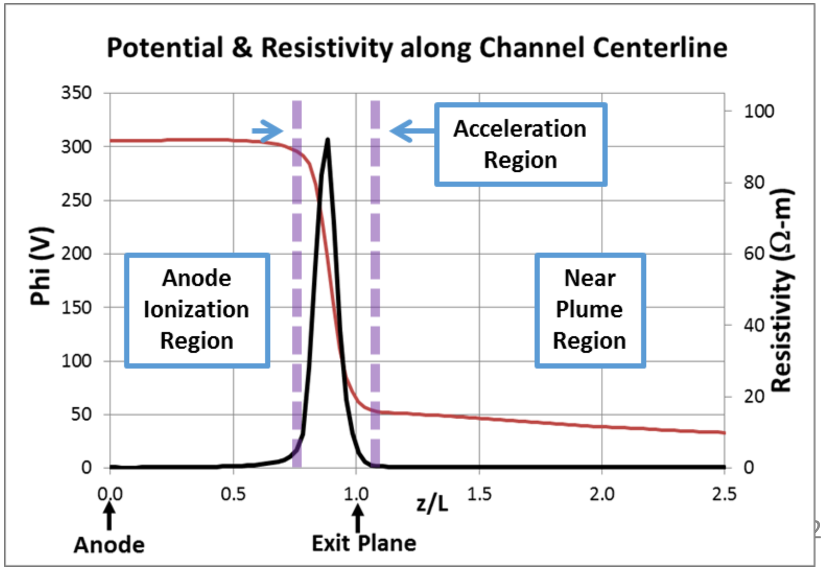


E_z

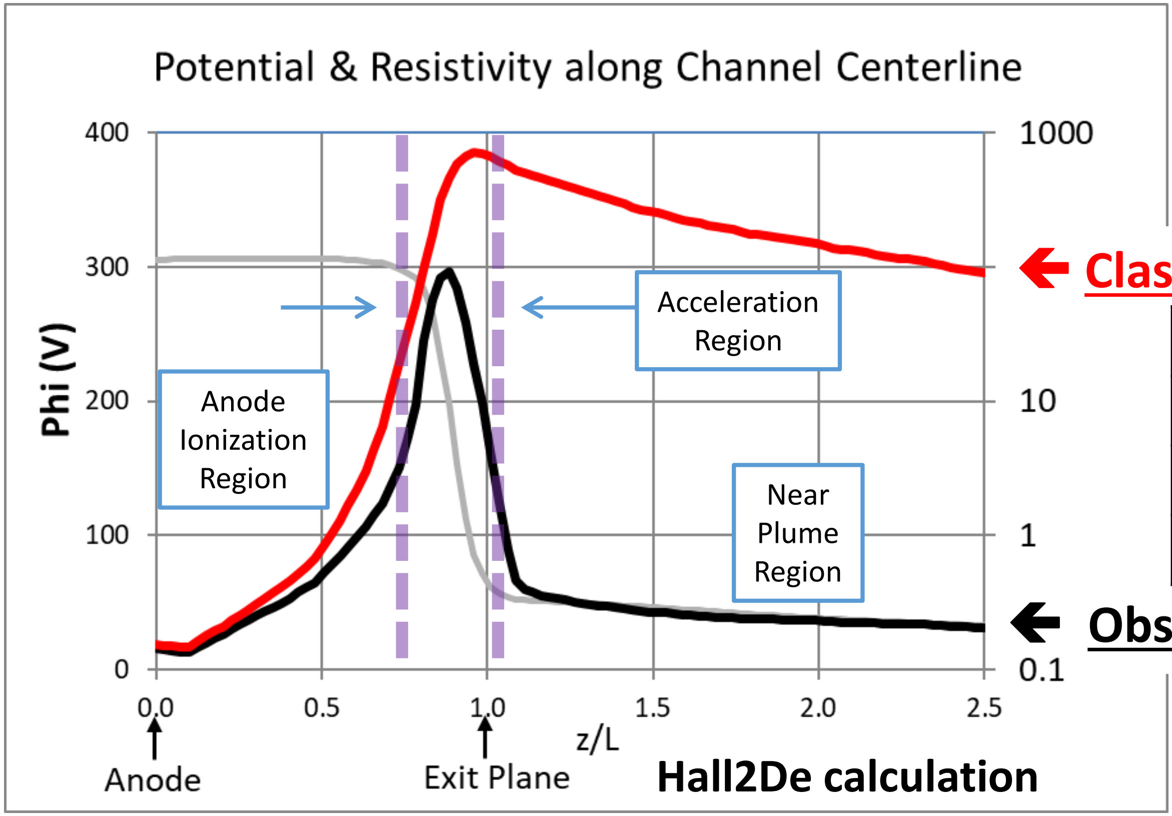
B_r

$$u_\theta = \frac{E_z}{B_r} = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

Hall2De calculation



Near Plume Observed Resistivity << Classical



Classical Resistivity

Observed Resistivity

Region	$\eta_{class}/\eta_{observed}$
Anode Ionization	**~1**
Acceleration	**~10-100**
Near Plume	**~1,000**

$$\eta_{class} = \frac{m_e \, \nu_e}{n e^2}(1 + \Omega^2)$$

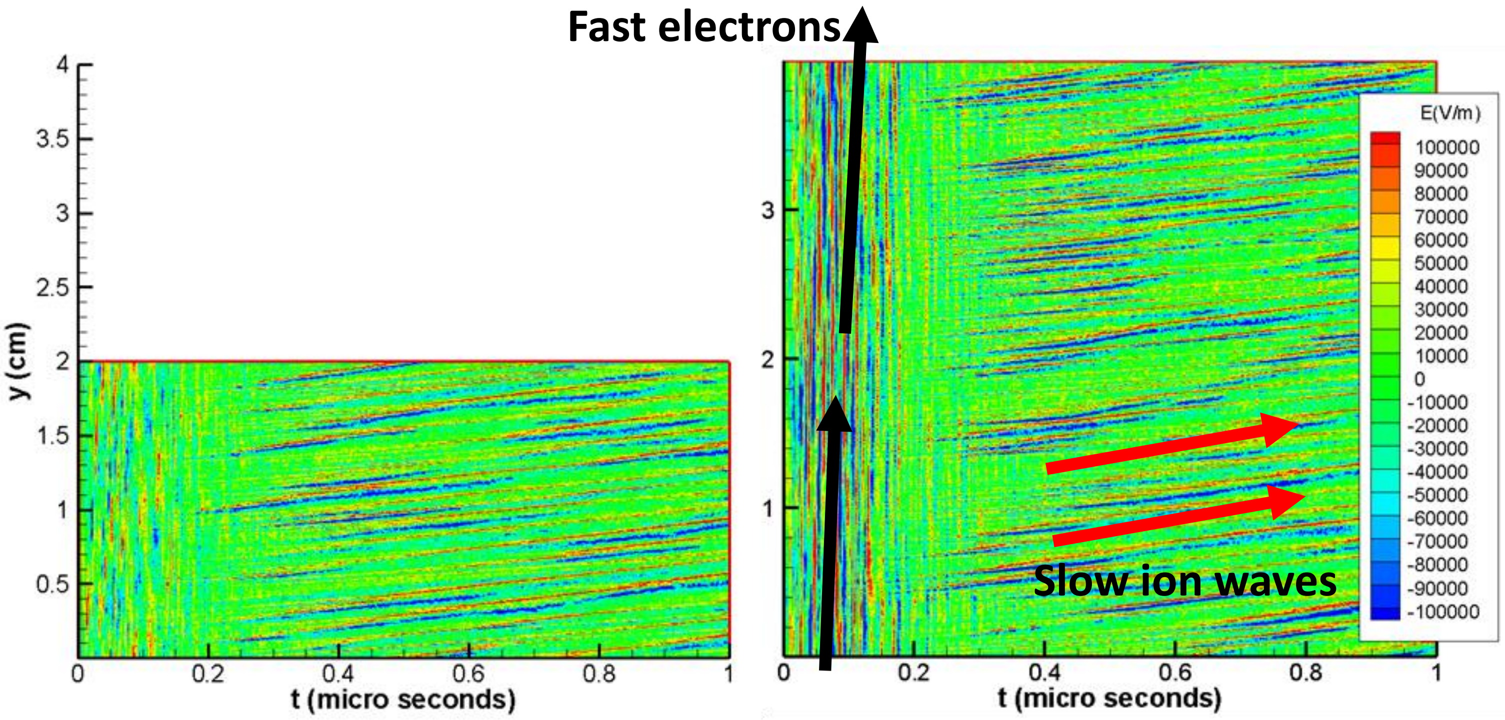
Hall parameter $\Omega \equiv \dfrac{\omega_{ce}}{\nu_e}$ $\quad \omega_{ce} = \dfrac{eB}{m_e}$

$$\boxed{\eta_{class} \sim \frac{B^2}{\nu_e}}$$

- Classical resistivity goes like B^2
- Observed Near Plume resistivity not increased by B
- Codes add “anomalous” scattering, ν_{anom}

1-D Azimuthal PIC

- 1-D PIC code
  - Solves Poisson's equation for potential, B_r = 200 G
  - Particle species: Electrons and Xe^+ ions
  - Periodic Boundary Conditions in azimuthal direction
- <u>Finding: ExB drifting electrons setup coherent ion waves</u>
  - Electrons transit grid many times before ion waves start to dominate
  - Consistent with closed drift electrons



Fast electrons

Slow ion waves

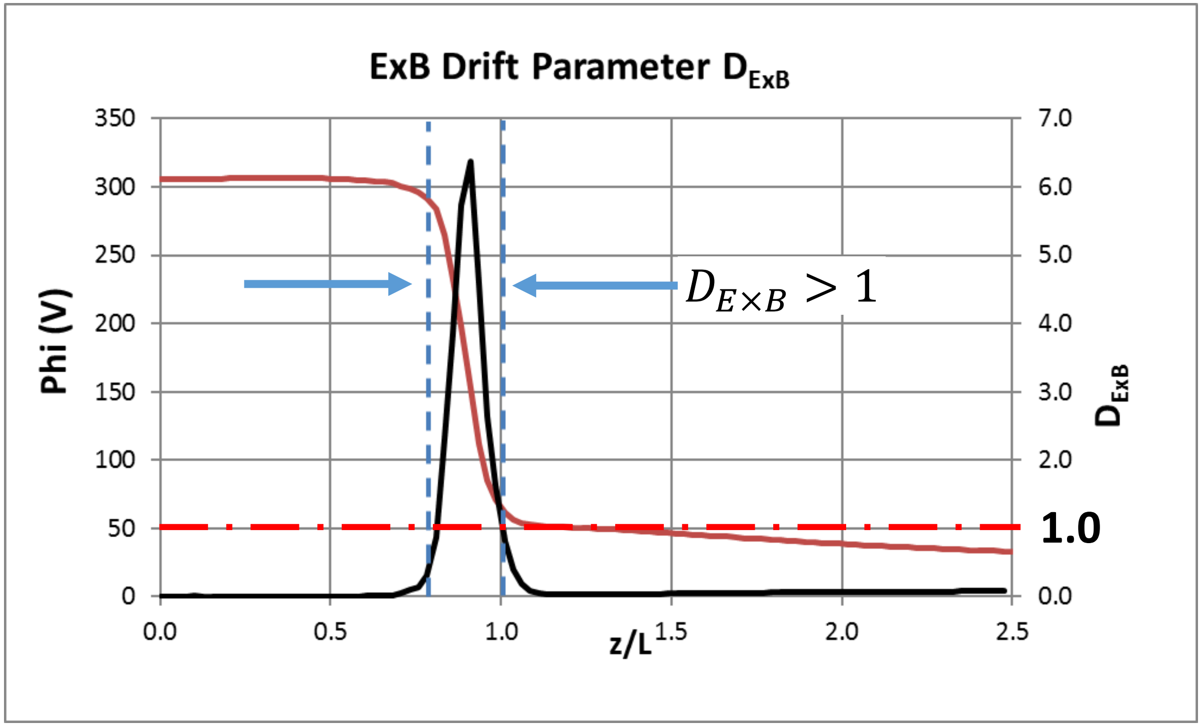
Hall Thruster Electron Drift Closed Only Near Acceleration Region

- ExB Drift Parameter: How many times around the channel before getting scattered

$$D_{E\times B} \equiv \frac{\tau_{scatter}}{\tau_{transit}} = \frac{u_{E\times B}}{2\pi\, R_{thruster}(\nu_{en}+\nu_{ei})}$$

Periodic Boundary Conditions probably not valid if $D_{E\times B} \ll 1$

Fourier transform ⇒ Periodic BC



ExB Drift parameter based on a Hall2De calculation

2-D PIC Domain based on $D_{E\times B}$

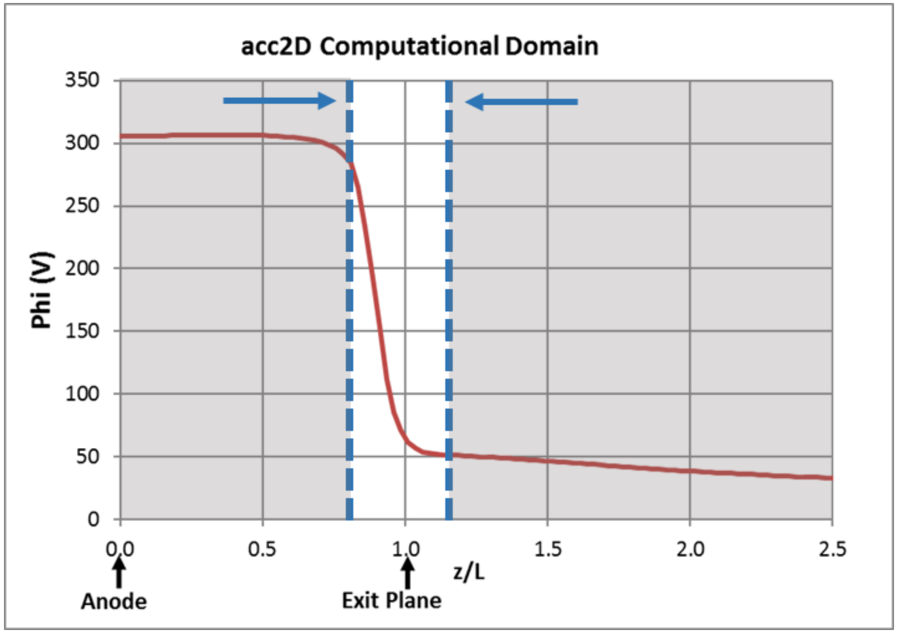
- 2-D azimuthal and axial electrostatic PIC code (acc2D)

$$m_e \frac{d\mathbf{u}_e}{dt} = -e(\mathbf{E} + \mathbf{u}_e \times \mathbf{B})$$

$$m_{Xe} \frac{d\mathbf{u}_{Xe}}{dt} = e\mathbf{E}$$

$$\nabla^2 \phi = -\frac{e}{\epsilon_0}(n_{Xe} - n_e)$$

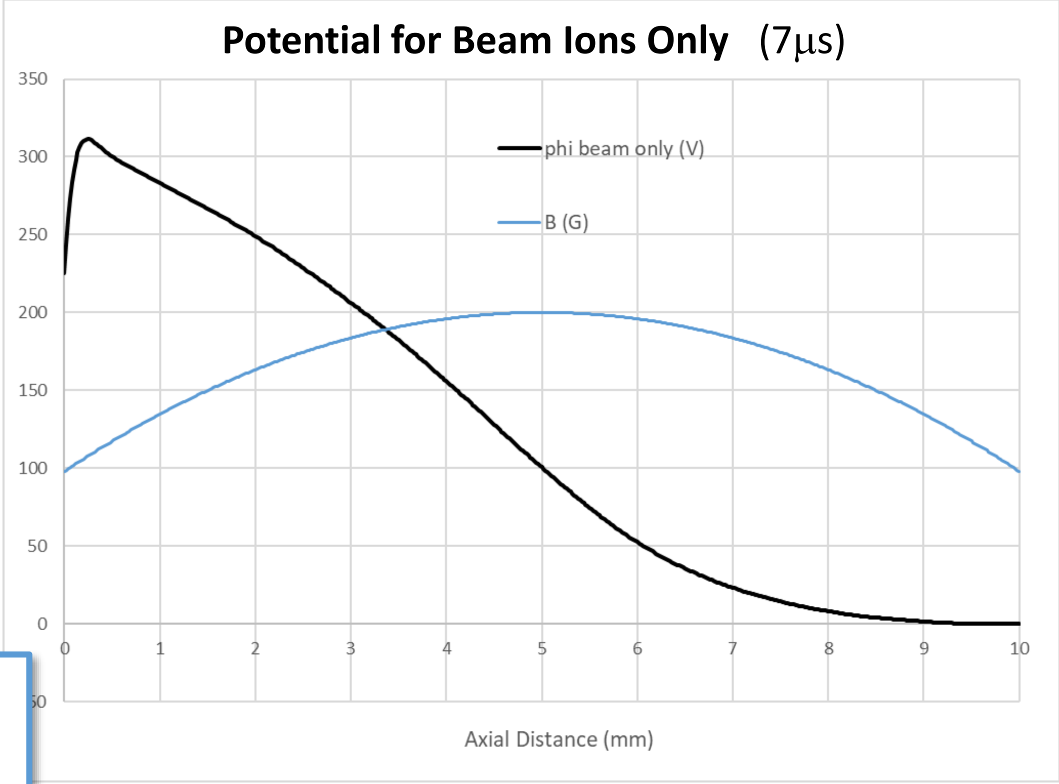
- Xe$^+$ ions enter upstream boundary: u_{i0} = 3000 m/s
- Electrons enter downstream boundary T_e = 2eV
- Typical run had 4 million macro particles



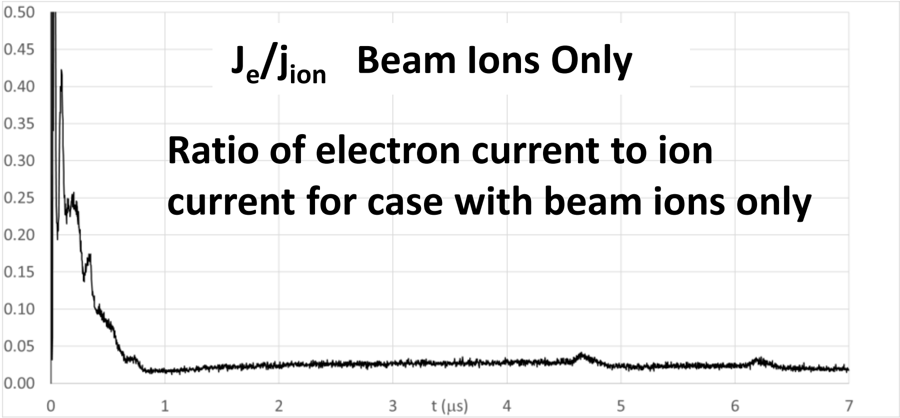
Parameter	Value
z length	10 mm
y length	2.5 mm
Nz	500
Ny	125
Δz, Δy	2×10^{-5} m
n_{i0}	4×10^{17} m^{-3}
u_{i0}	3000 m/s (6.2 eV)
j_{i0}	200 A/m^2
T_{i0}	0.1 eV
T_{e0}	2 eV
NG	32
Δt	2.5×10^{-12} s
φ_{anode}	225 V
$\varphi_{cathode}$	0 V
B_{max}	200 G
B_{min}	100 G

2-D Results: Beam Ions Only

- Quasi-steady state in ~1μs
- Very broad potential profile
  - Spans the computational domain
- Very little electron current
  - J_e/jion = 2.5%
  - Should be ≈ 30%
- **Why so little electron transport?**


Potential for Beam Ions Only (7μs)

- A feedback mechanism in 1-D simulations unphysically increases ion acoustic wave amplitudes.
- Ions entering the grid upstream immediately feel wave electric fields from ions leaving the grid.
- This feedback is not in Hall thrusters, where ion acoustic electric fields convect with the ions.

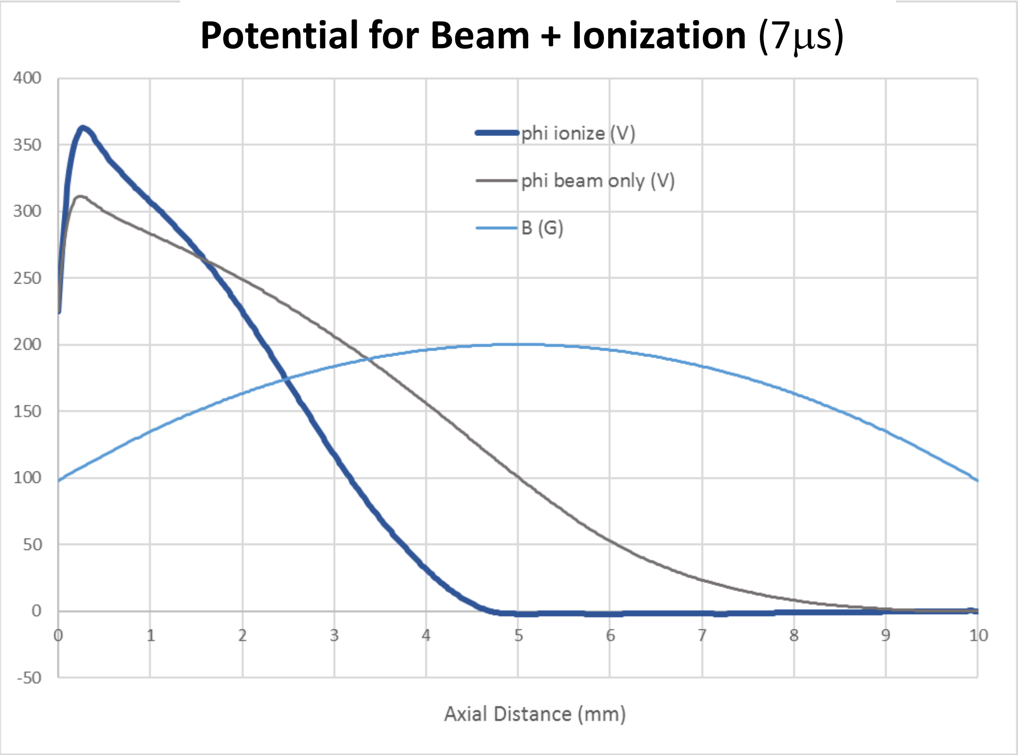

J_e/j_{ion} Beam Ions Only

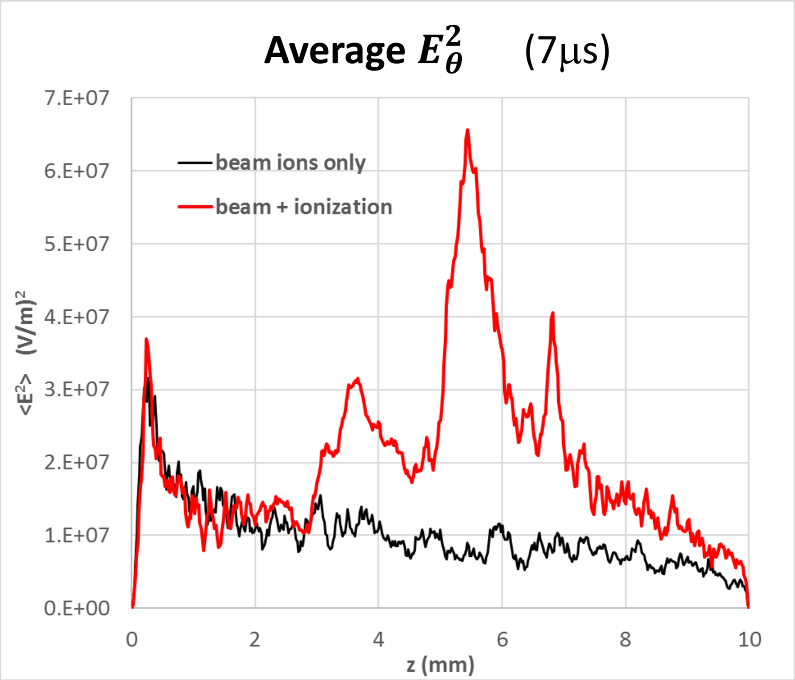
Ratio of electron current to ion current for case with beam ions only

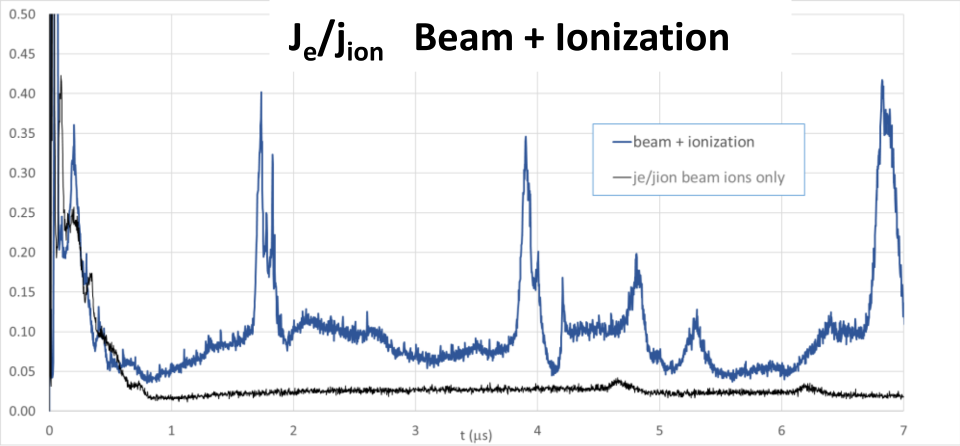
2-D Results: Beam + Ionization

- Small, realistic, amount of ionization
  - ~6% of beam current
- Two potential zones
  - Steep acceleration region
  - Flat near plume region
- Electron current - 4X beam only

  J_e/j_{ion} = 10%
- Downstream $\langle E_\theta^2 \rangle$~ 3X beam only







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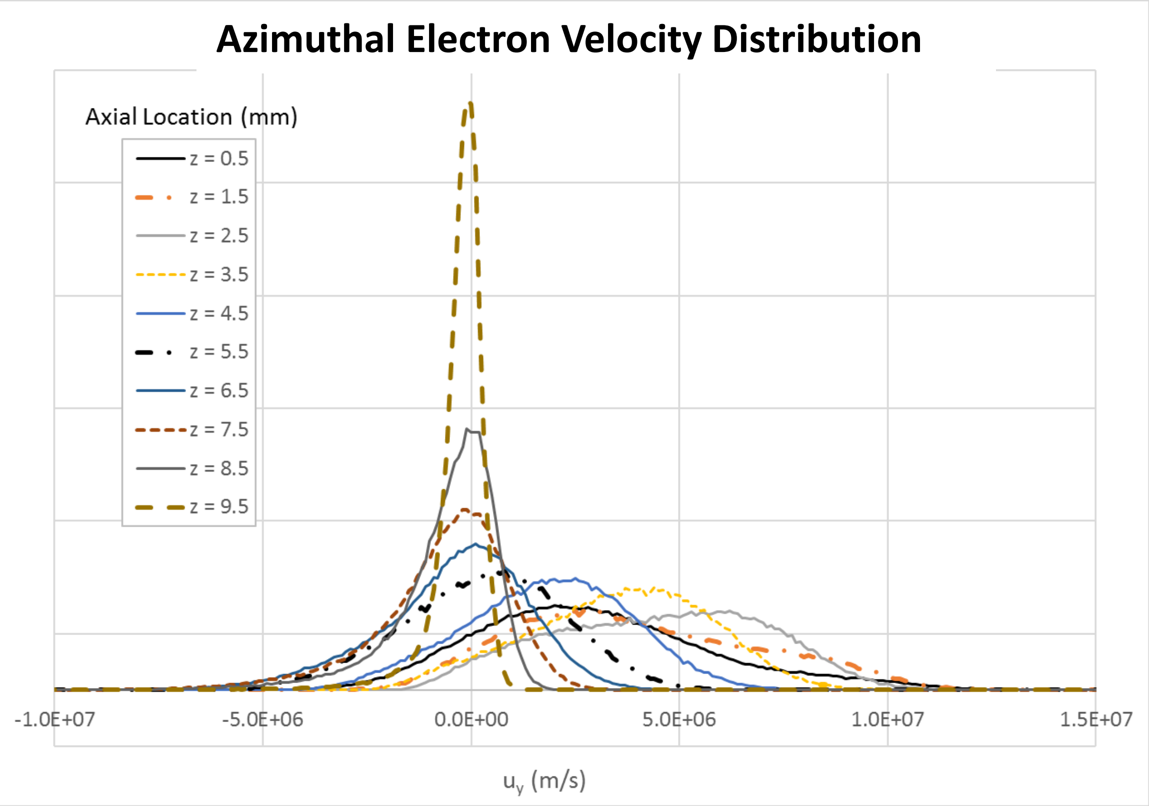
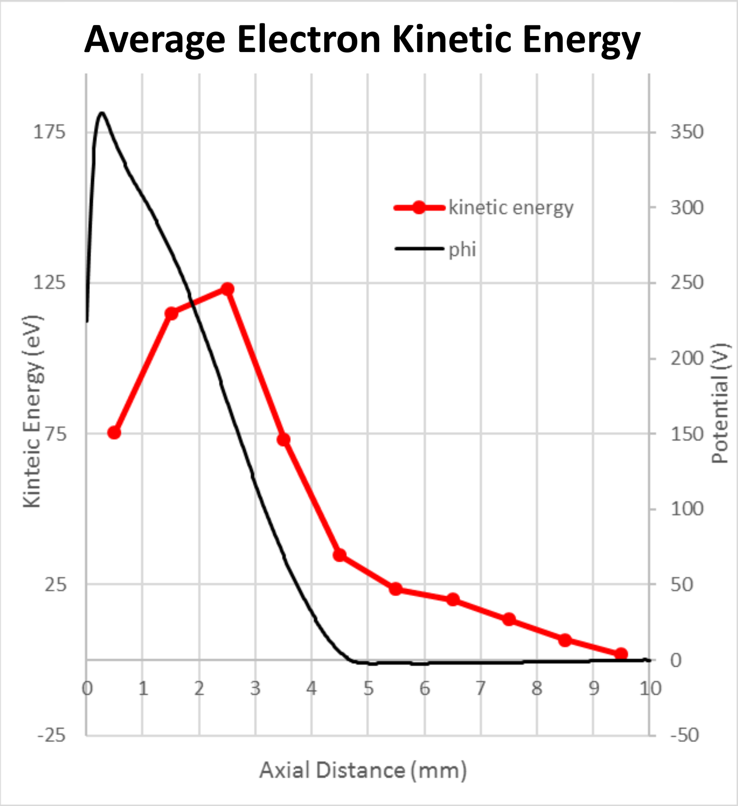
Electron Velocities & Energies

- Electron distributions become less Maxwellian as electrons move upstream
- Electron kinetic energy increases rapidly in high electric field region


Azimuthal Electron Velocity Distribution


Average Electron Kinetic Energy

Downstream Electrons Ignore the **B** Field - Why?

- Classical – Hall parameter is a measures importance of magnetic field

$$\Omega_{class} \equiv \frac{\omega_{ce}}{(\nu_{en} + \nu_{ei})} = \frac{eB}{m_e(\nu_{en} + \nu_{ei})}$$

- PIC code – Magnetic field enters only through electron Lorentz force

$$m_e \frac{d\mathbf{u}_e}{dt} = -e(\mathbf{E} + \mathbf{u}_e \times \mathbf{B})$$

- In azimuthal direction, electric field is from waves

$$m_e \frac{du_\theta}{dt} = -e(E_{wave} + u_z B)$$

- Define: <u>Anomalous Hall parameter</u> ⇒ measures importance of magnetic field

$$\Omega_{anom} \equiv \frac{u_z B}{E_{wave}}$$

- For a given wave amplitude, E_{wave}

<u>slower electrons are unmagnetized</u>

$$\Omega_{anom} = \frac{u_z B}{E_{wave}} < 1\,, \qquad u_z < \frac{E_{wave}}{B}$$

<u>faster electrons are magnetized</u>

$$\Omega_{anom} = \frac{u_z B}{E_{wave}} > 1\,, \qquad u_z > \frac{E_{wave}}{B}$$

Enough Cold Electrons Downstream to Carry the Current

- Axial drift velocity $\overline{u_z}$

$$\overline{u_z} = \int_0^{u_1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u_x, u_y, u_z)\, u_z \, du_x du_y du_z$$

- $u_1 = \sqrt{\dfrac{8\; e\; T_e}{m_e}\dfrac{\overline{u_z}}{\bar{c}}}$

- Azimuthal Electric Field in downstream half of the domain

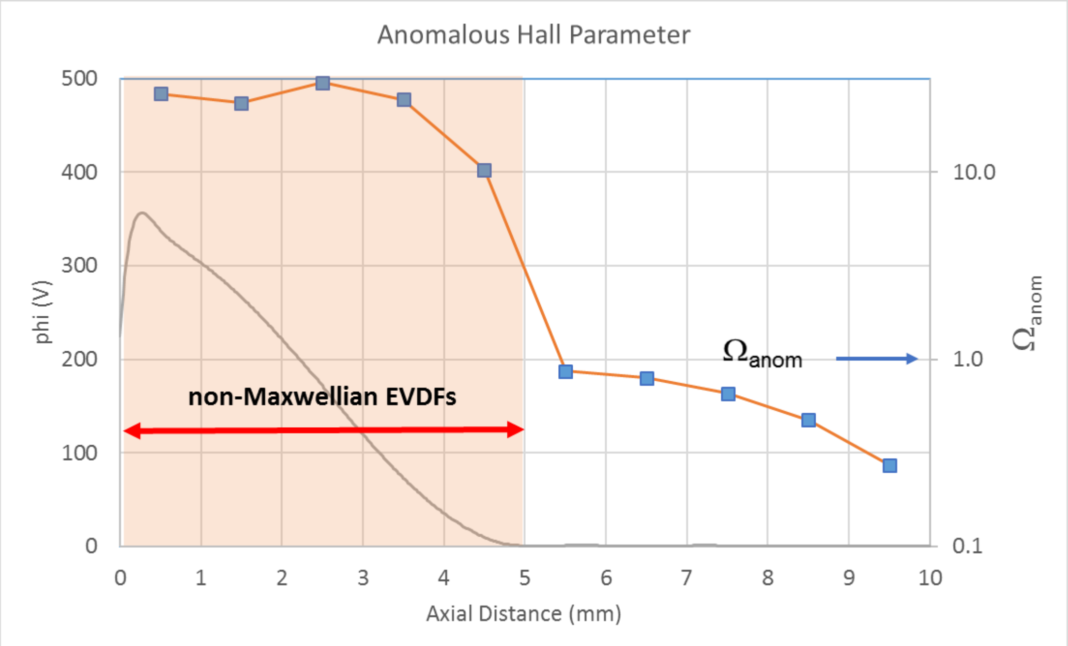
$$\sqrt{\langle E_y^2\rangle} = 3500\,\frac{V}{m}\;.$$

- Downstream: Azimuthal waves “unmagnetize” enough cold electrons to carry the current

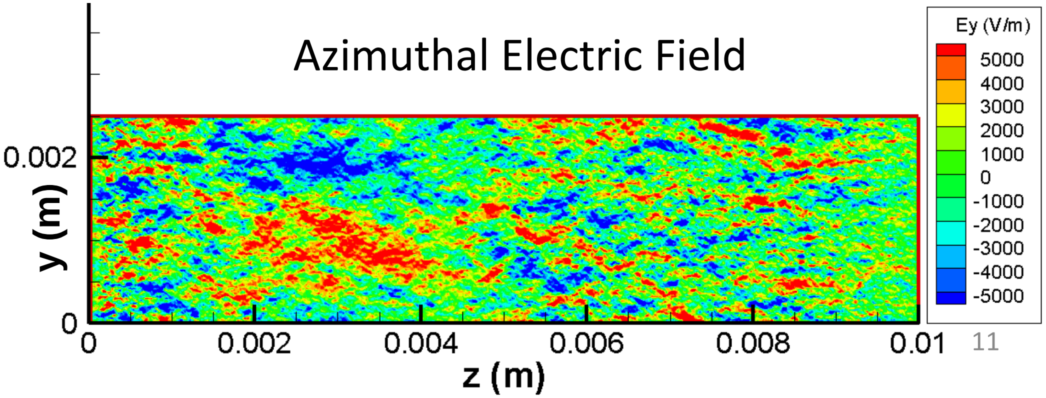
$$\Omega_{anom} = \frac{u_1 B}{E_{wave}} < 1$$

- Upstream: Electrons have higher velocities and are magnetized

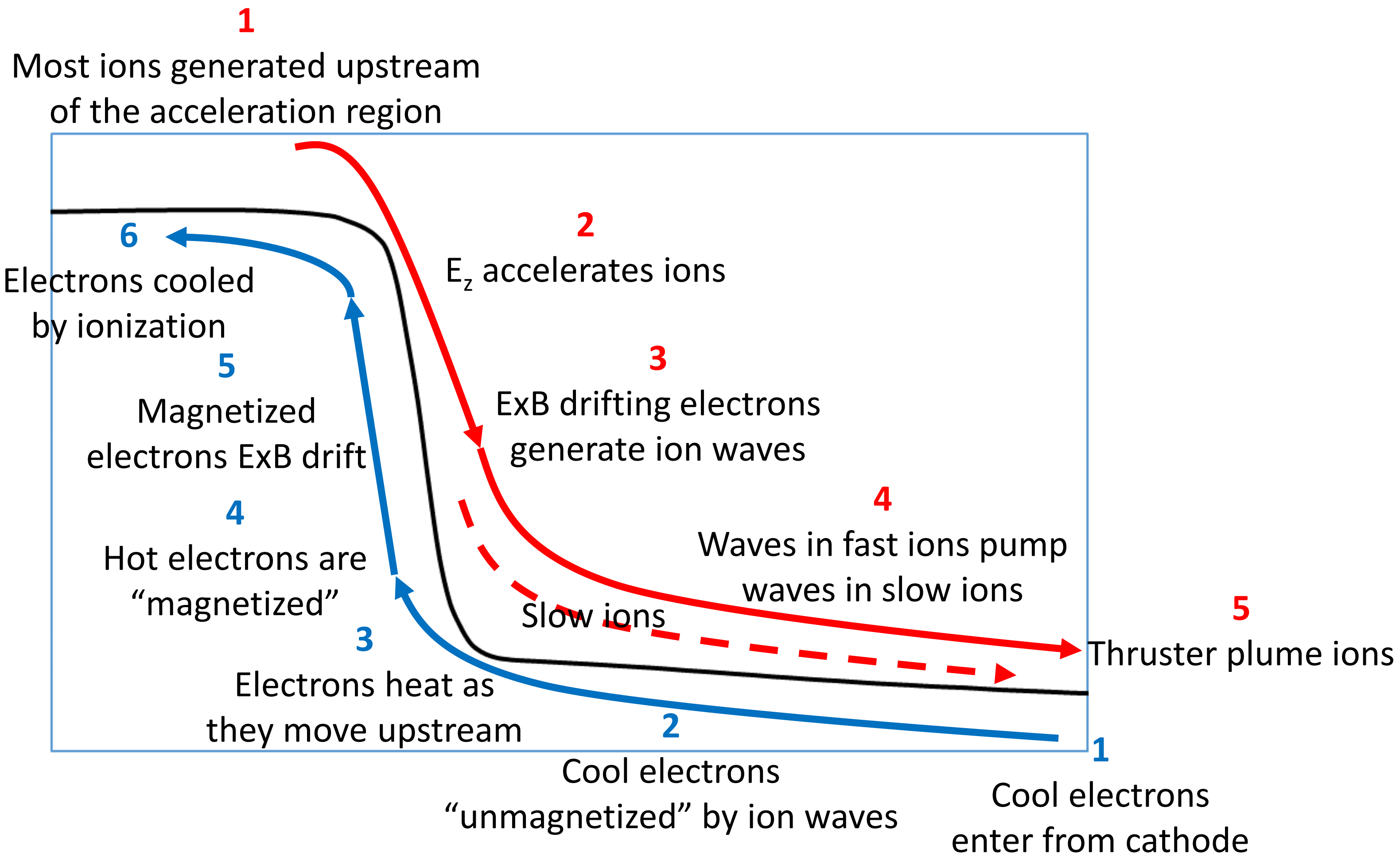
$$\Omega_{anom} = \frac{u_1 B}{E_{wave}} > 1$$



Anomalous Hall parameter calculated assuming the EVDFs are Maxwellian in the downstream half of the computational domain. The ExB drift velocity dominates upstream. Also shown is the calculated potential profile.



Hall Thruster Charged Particle Transport
"A Hypothesis"



1
Most ions generated upstream
of the acceleration region

2
E_z accelerates ions

3
ExB drifting electrons
generate ion waves

4
Waves in fast ions pump
waves in slow ions

Slow ions

5
Thruster plume ions

1
Cool electrons
enter from cathode

2
Cool electrons
"unmagnetized" by ion waves

3
Electrons heat as
they move upstream

4
Hot electrons are
"magnetized"

5
Magnetized
electrons ExB drift

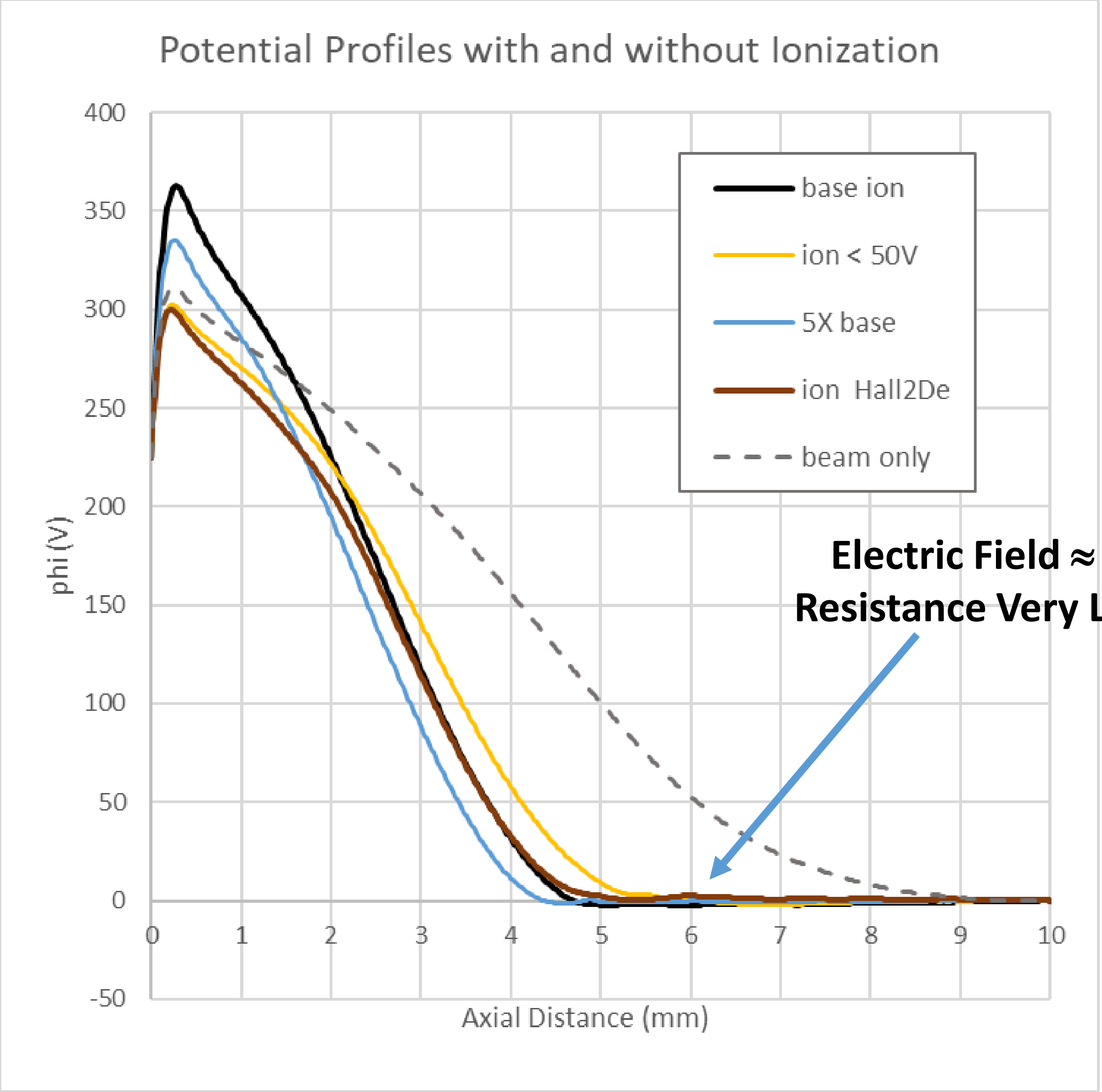
6
Electrons cooled
by ionization

Conclusions

- In most of the Hall thruster, electrons are scattered many times before they drift around the channel ($D_{ExB} < 1$)
  - Acceleration region is the exception
  - Quasi-linear theory assumes periodic BC
    - strictly true only in acceleration region
- 2-D z-θ simulation domain chosen so $D_{ExB} \geq 1$
  - Based on Hall2De calculation
  - Acceleration region and beginning of the near plume region
- Beam ions only - little electron transport
- Realistic levels of ionization
  - Much greater electron transport with
  - Flat potentials profile in “near plume” region
- Anomalous Hall parameter $\Omega_{anom} \equiv \dfrac{u_z B}{E_{wave}}$
  - Slower electrons act as if unmagnetized $\Omega_{anom} < 1$
  - Faster electrons are magnetized $\Omega_{anom} > 1$

Backup

Downstream Potentials Flat with Ionization



Electric Field ≈ 0
Resistance Very Low

Anomalous Hall Parameter

- Axial drift velocity $\overline{u_z}$
- Maxwellian distribution $f(u_x, u_y, u_z) = \left(\frac{m}{2\pi e T_e}\right)^{\frac{3}{2}} exp\left(-\frac{m}{2eT_e}(u_x^2 + u_y^2 + u_z^2)\right)$
- $\overline{u_z} = \int_0^{u_1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(u_x, u_y, u_z) u_z \, du_x du_y du_z$
- $\overline{u_z} = \frac{\bar{c}}{4}\left(1 - \exp\left(-\frac{\mathcal{E}}{T_e}\right)\right) \approx \frac{\bar{c}}{4}\frac{\mathcal{E}}{T_e}$
- $\bar{c} = \sqrt{\frac{8\, e\, T_e}{\pi\, m_e}}$
- $\mathcal{E} \equiv \frac{1}{2}\frac{m_e}{e}\, u_1^2$
- $u_1 = \sqrt{\frac{8\, e\, T_e}{m_e}\frac{\overline{u_z}}{\bar{c}}}$

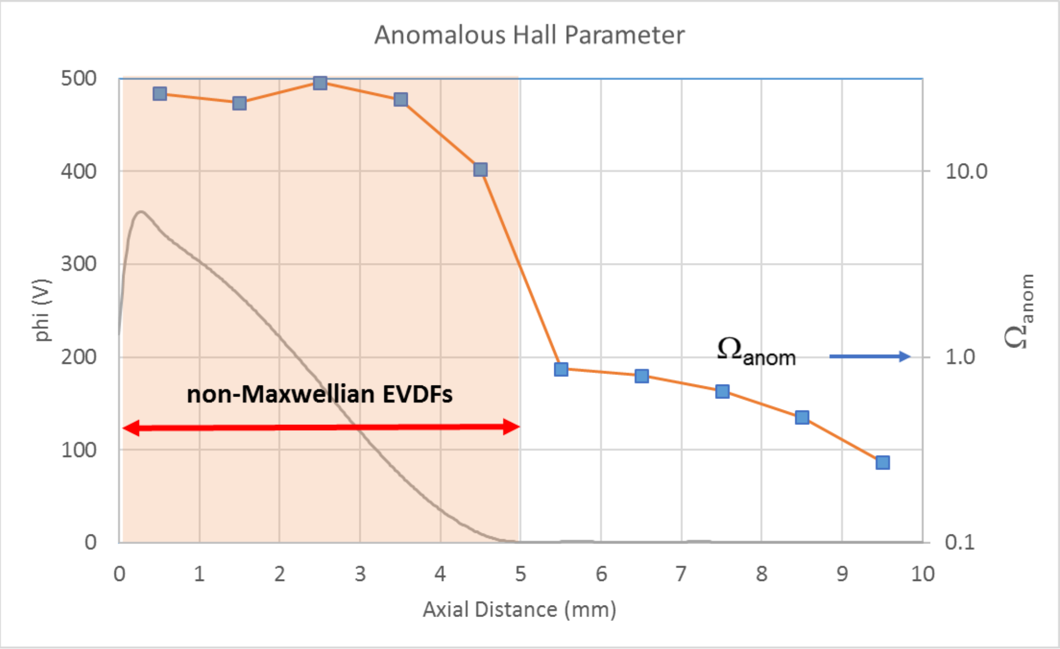
Anomalous scattering from quasi-linear theory not consistent with observed sudden change in resistivity

$$\nu_{anom}^{quasi-linear} = \frac{\omega_{pe}}{T_e^2}\sum_k (e\phi_k)^2$$

Maxwellian Anomalous Hall

z (mm)	T_e (eV)	u_{beam} (m/s)	$\overline{u_z}$ (m/s)	u_1(m/s)	B (G)	E_{wave}/B (m/s)	E_z/B (m/s)	Ω_{anom}
0.5	75	4.8E+03	4.8E+02	9.3E+04	119	2.9E+05	7.7E+06	26.3
1.5	115	1.2E+04	1.2E+03	1.6E+05	151	2.3E+05	5.4E+06	23.4
2.5	123	1.7E+04	1.7E+03	2.0E+05	175	2.0E+05	6.0E+06	30.1
3.5	73	2.0E+04	2.0E+03	1.9E+05	191	1.8E+05	4.5E+06	24.4
4.5	35	2.2E+04	2.2E+03	1.7E+05	199	1.8E+05	1.81+06	10.3
5.5	24	2.3E+04	2.3E+03	1.5E+05	199	1.8E+05		0.9
6.5	20	2.3E+04	2.3E+03	1.5E+05	191	1.8E+05		0.8
7.5	14	2.2E+04	2.2E+03	1.3E+05	175	2.0E+05		0.7
8.5	6.6	2.2E+04	2.2E+03	1.1E+05	151	2.3E+05		0.5
9.5	1.9	2.2E+04	2.2E+03	8.0E+04	119	2.9E+05		0.3

Anomalous Hall parameter calculated assuming the EVDFs are Maxwellian in the downstream half of the computational domain and the ExB drift velocity dominates upstream. Also shown is the calculated potential profile.

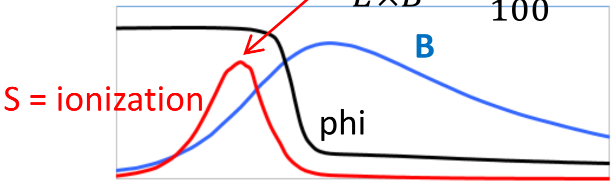


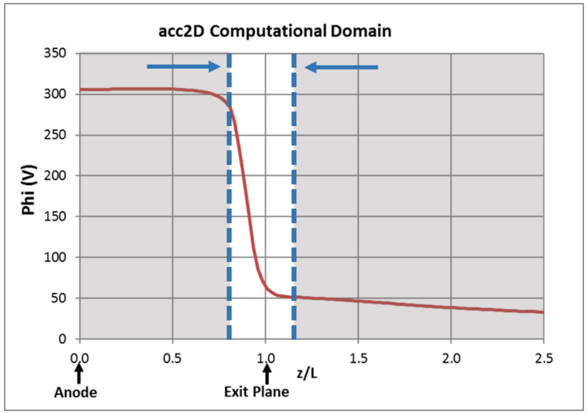
Comparison with Boeuf & Garrigues

This Simulation

- Domain $D_{ExB} > 1$
- Includes small fraction of ionization
- B_{max} = 200 G
- Maximum $E_{wave} \approx 10^4$ V/m
- Potential profiles similar to Boeuf
- J_e/j_{oin} small

$$D_{E\times B} \approx \frac{1}{100}$$

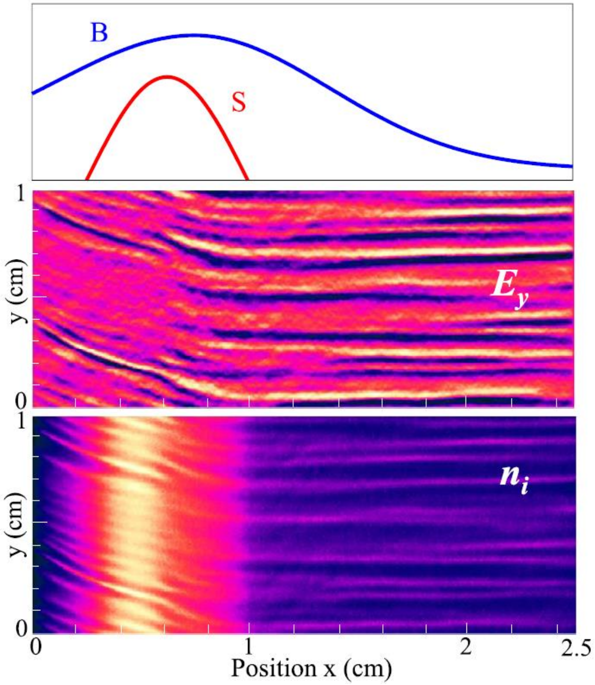




Boeuf & Garrigues

"E × B electron drift instability in Hall thrusters: Particle-in-cell simulations vs. theory", Physics of Plasmas 25, 061204 (2018)

- Domain includes all ionization
- Periodic BC allows waves everywhere
- B_{max} = 100 G
- Maximum $E_{wave} \approx 5\text{x}10^4$ V/m
- Large E_{wave} upstream of ionization peak
- J_e/j_{oin} large



Acknowledgements

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